various kinds of functions, Banach algebras, and function algebras) while the composition operation attracts much less attention.

This book is devoted to the study of the so-called linear superpositions, which were motivated by Arnold and Kolmogorov's solution of Hilbert's 13th problem.

Let $X = X_1 \times X_2 \times \cdots X_k$ (compact factors) and let Q be a closed subset of X. Let $D = D(Q)$ denote the linear subspace of $C(Q)$ which consists of the functions of the form

$$f(x) = g_1(x_1) + g_2(x_2) + \cdots + g_k(x_k), \qquad x = (x_1, x_2, ..., x_k) \in Q,$$

with $g_i \in C(X_i)$, $i = 1, 2, ..., k$. Q is said to be *basically* embedded in X if $D = C(Q)$. The first chapter in the book is devoted to the study of this property. It includes Kolmogorov's Superposition Theorem, characterization of basic embeddings in terms of separation of measures, the study of the dual to the linear superposition operator, characterization of the dimension of Q by basic embeddings and related topics.

In Chapters 2 and 3 it is assumed that D differs from $C(Q)$. The following questions are then asked.

- When is D dense in $C(Q)$?
- When is D closed in $C(Q)$?
- Given an element f of $C(Q)$, what is the distance of f from D?
- Is this distance attained by some element of D?
- Is there an algorithm to find the nearest element?

Chapter 2 is devoted to the approximation of functions of two variables by sums of the form $g(x) + h(y)$. This problem was first studied in 1951 by Diliberto and Straus who developed the main tools for it (finitely supported annihilating measures on one hand and the leveling algorithm on the other). Several authors have extended their work. Some false results were also published, mainly when it was carelessly assumed that the case of three or more variables could be handled like the two-variable case. These topics are either studied or referred to in Chapter 2.

In Chapter 3 some more general forms of linear superpositions are developed.

The author avoids the presentation of proofs of some deeper and more difficult results such as the characterization due to Marshall and O'Farrell of extreme annihilating measures and my characterization of dimension by linear superpositions and basic embeddings. Some recent developments of the theory are omitted. These are mainly topologically oriented results such as Skopenkov's characterization of $R \times R$ basically embeddable continua, the case of one-dimensional factors X_i, in particular, dendrites, and hereditarily indecomposable X_i.

I regard this book by Khavinson as a solid introduction to the theory of linear superpositions.

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S. Bagdasarov, *Chebyshev Splines and Kolmogorov Inequalities*, Operator Theory: Advances and Applications **105**, Birkhäuser, Basel, 1998, xiii + 207 pp.

The study of extremal problems in approximation theory has a long and distinguished history. A. N. Kolmogorov had a significant influence on the study of such problems via two seminal papers from the 1930's. In the first of these papers Kolmogorov introduced the concept of n-widths. The second paper provided a solution for what we now call the Kolmogorov–Landau problem on all of $\mathbb{R}$. In considering both of these extremal problems in the Sobolev spaces $W_{\infty}^{(n)}[a, b]$ and their periodic analogues $\tilde{W}_{\infty}^{(n)}$, an exceptional role is played by the polynomial perfect splines of degree n, i.e., those functions whose nth derivatives take, on intervals, the values c and $-c$.

This research monograph considers certain generalizations of these two problems and their associated extremal functions. The function classes considered are $W^rH^{\omega}[a, b]$ and $\widetilde{W^rH^{\omega}}$ for certain sets of ω. These are defined in the following manner. Let $\omega(f; \cdot)$ denote the modulus of continuity of f, and let ω denote any fixed concave modulus of continuity. Then

$$W^rH^{\omega}[a, b] = \{f : f^{(r)} \in C[a, b],\ \omega(f^{(r)}; t) \leqslant \omega(t),\ t \in [0, b-a]\},$$

while

$$\widetilde{W^rH^{\omega}} = \{f\colon f \in W^rH^{\omega}(\mathbb{R}),\ f\ 2\pi \text{ periodic}\}.$$

For the case $\omega(t) = t$ we have $W^rH^{\omega}[a, b] = W_{\infty}^{(r+1)}[a, b]$ and $\widetilde{W^rH^{\omega}} = \tilde{W}_{\infty}^{(r+1)}$. These function classes were introduced by S. M. Nikol'skii in the 1940s. They have been studied by N. P. Korneichuk and others.

To quote the author, the three main aims of this book are "(1) to introduce the notion and give the formulae for the perfect ω-splines in W^rH^{ω}; (2) to describe various extremal properties of perfect ω-splines by emphasizing the new phenomena and the old features inherited from polynomial perfect splines; and (3) to show examples of applications of the general theory of perfect splines in examples related to the computation of n-widths of classes $W^rH^{\omega}(I)$ and our solution of one of the most celebrated problems of real analysis—the Kolmogorov problem of sharp inequalities for intermediate derivatives in the Hölder classes $W^rH^{\alpha}(\mathbb{R}_+)$ and $W^rH^{\alpha}(\mathbb{R})$."

This book contains 17 chapters and 2 appendixes. Chapters 0–3, 6–9, and 14 contain the introduction, a review of known facts, and a proof of the main results concerning Chebyshev ω-splines. In Chapters 4, 5, and 10–12 and Appendixes A and B we find applications of these results to various forms of the Kolmogorov–Landau problem. Chapters 13, 15, and 16 contain solutions of the problem of n-widths for various such spaces.

This is a research monograph and not a textbook. As such it is detailed and not always easy to follow. However, it contains a wealth of information and is a must for any researcher in this field.

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Han-lin Chen, *Complex Harmonic Splines, Periodic Quasi-Wavelets. Theory and Applications*, Kluwer Academic, Dordrecht, 2000, xii + 226 pp.

The following review is taken from the preface of the book, with permission from the publisher.

This book, written by our distinguished colleague and friend, Professor Han-lin Chen of the Institute of Mathematics, Academia Sinica, Beijing, presents, for the first time in book form, his extensive work on complex harmonic splines with applications to wavelet analysis and the numerical solution of boundary integral equations. Professor Chen has worked in approximation theory and computational mathematics for over forty years. His scientific contributions are rich in variety and content. Through his publications and his many excellent Ph.D. students he has taken a leadership role in the development of these fields within China. This new book is yet another important addition to Professor Chen's quality research in computational mathematics.